CAN PROBABILITY BE SUBJECTIVE AND OBJECTIVE AT THE SAME TIME? A REPLY TO ARNOLD BAISE

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Introduction

THE CENTRAL ARGUMENT that I have made over the last several years on these pages and elsewhere (2009a, 2009b, 2010, 2011a, 2011b) is that probability is a subjective measure of human belief, not a measure of an "objective" property of things or events in the world. My argument was primarily directed at those members of the Austrian School who have adopted and defended the frequentist definition of probability articulated by Ludwig von Mises's brother, Richard von Mises's. According to Richard von Mises, probability is an "objective" property of things in the world. Just as things have a weight and a length, he held, they have a physical property of probability "in them" that can be measured. My papers primarily aimed to demonstrate that Richard von Mises's definition and conception of probability as an "objective" physical property "out there" in the world is completely inconsistent with the conception of causality held by most members of the Austrian School.

My claim that probability ought to be defined as a purely subjective measure of human belief has been challenged in a recent and interesting article on these pages by Arnold Baise (2011). Baise argues that probability ought to be defined, not as a purely subjective measure of human belief, as I have claimed, but rather in the following way:

> Probability P(A | I) is a number between 0 and 1 that indicates how plausible it is that proposition A is true, based on information I. In addition, one could add that a probability of 1 indicates certainty that the proposition is true, while a probability of 0 indicates certainty that the proposition is false. (2011, p.3).

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The reasoning that leads Baise to advance this definition for probability, however, is seriously and apodictically flawed. As a consequence, his definition for probability must be rejected as a viable alternative to my purely subjective definition.

Muddying the Water

The first problem with Baise's article is that it muddies the water surrounding this particular definitional debate. I say this, in the first place, because it is not exactly clear what purpose the article is intended to serve. On the one hand, the fact that Baise advances a completely different definition for probability than both Richard von Mises and myself leads one to believe that he is trying to join the definitional debate. On the other hand, the fact that he chooses not to discuss or critique either Richard von Mises's definition or my definition leads one to think that he is trying to keep out of the debate. After all, how can one join a definitional debate without critiquing the existing definitions, or, at the very least, explaining where one's definition fits into the existing literature? Strangely, it is up to the reader to divine the purpose of the article.

This might not be such a serious oversight, except that Baise claims to be offering an "alternative approach to understanding probability" (2011, p.3.) than the purely subjective approach that I have defended. As such, one would expect that he would examine and critique the argument that I have made for a purely subjective definition. This would put him in a position to say: "Here is Crovelli's argument, this is why he is wrong, and this is why we need an alternative definition." No such analysis is provided, however, which leaves the reader completely in the dark about why Baise thinks it necessary to advance an "alternative approach" in the first place. He must have a reason for rejecting my argument for a purely subjective definition and offering an alternative, but he provides no discussion or critique of my argument whatsoever.

For these reasons, Baise's article muddies the definitional water more than it clarifies anything. The problems with Baise's article go far deeper than merely muddying the definitional water, however. This sections that follow are devoted to highlighting some of the more serious errors in reasoning contained in the article.

What Exactly Does Probability Measure?

Before proceeding, it is important to highlight the critical difference between the purely subjective definition for probability that I have defended and the "objective Bayesian" definition that Baise has advanced. The

subjective definition for probability that I have defended asserts that probability is a subjective *measure* of human uncertainty. That is, probability is used as a *measure* of how uncertain man is that events will or will not occur. Baise's definition for probability, on the other hand, asserts that while probability "describes" something inside man's mind, it is nevertheless "objective" in the sense that men "must" assign the same probability to an event *if they have the same prior information*. In other words, while Baise is willing to stipulate that probability "describes something inside the mind" (2011, p.2), he is trying to bring objectivity in the back door by claiming that men *ought* to assign the same probability to an event if they have the same prior information.

Bearing this important difference in mind, the first thing that should be noted is that Baise is extremely unclear about what probability is used to measure.[1] He obviously does not agree with the idea that probability measures human uncertainty, as I have claimed, or else he would not have written the article in the first place, but beyond that things get quite fuzzy. For example, does he mean that probability is used to measure what "ought to be" in man's mind when we say that the probability of rain tomorrow is 50%? If so, what does that even mean? How can you measure what "ought to be" in man's mind? Moreover, saying that we are measuring what "ought to be" in man's mind necessarily implies that what man *does in fact* believe is often different from what he *ought* to believe. This means we are *not* measuring something that is subjective in the epistemological *or* the metaphysical sense, because we are no longer measuring something "in man's mind." Instead, we are measuring something that "ought to be" in his mind, *but may or may not actually be there*. In other words, we are back to an objective definition for probability akin to Richard von Mises's definition.

Alternatively, is Baise claiming that are we measuring the truth of a proposition when we say that the probability of throwing a two with a die is 1/6? If so, then why are we using a numerical scale?[2] Propositions are either true or false—you do not need or even want a finely-graded scale to say whether a proposition is true or false. In order to determine the truth value of a proposition you only need two words: *True* and *False*. The numerical scale that is conventionally used in probability (i.e., 0 to 1) is only necessary or useful if we are measuring *man's uncertainty or beliefs* about the truth or falsity

[1] On probability as a "measure number," see Robert A. Crovelli "An Analysis of the Basic Concepts in Applied Mathematics," *Mises Working Papers Series*, p. 22.

[2] It is important to note that the idea that probabilities must be a number between 0 and 1 is highly debatable, and depends on the definition we adopt for probability. Baise, however, simply takes it for granted that probabilies must be a number between 0 and 1. For an argument that probabilities need not be numbers, see M. Crovelli (2010).

of a proposition, but that takes us back to the purely subjective definition. In addition, if probability is taken to be a measure of the truth of propositions, then how is Baise's definition any different than the so-called "A priori theory" of probability defended by John Maynard Keynes or the so-called "Classical" or "Combinatorial" theory of probability?

This lack of clarity about what probability is used to measure is not resolved by the definition for probability that Baise provides at the end of the article. Again, his definition asserts that:

> Probability P(A|I) is a number between 0 and 1 that indicates how plausible it is that proposition A is true, based on information I. In addition, one could add that a probability of 1 indicates certainty that the proposition is true, while a probability of 0 indicates certainty that the proposition is false. (2011, p.3).

First of all, this definition does not assert that probability has anything to do with what man "ought to" believe, which is rather curious given the thrust of the rest of the article. Nor does it assert that men "must" assign the same probability to an event if they have the same prior information, which is even more curious, given the numerous citations of E.T. Jaynes in the article. In fact, the first sentence of this definition can be interpreted as affirming the subjective definition for probability, since the word "plausible" is virtually synonymous with the word "believable," Hence, Baise's definition for probability does not clarify the vital question of what probability measures.

The reader is thus left to wonder: If probability is not used to measure something "out there" in the world, and it is not used to measure human uncertainty, then what on Earth is it used to measure? If this question cannot be adequately answered, and it is clear that Baise has not adequately answered it, then it is quite a stretch to say that he has provided us with a definition for probability *at all*, let alone a definition that improves on either Richard von Mises's definition or my definition.

How Can Men Have Exactly the Same Prior Information?

The next important issue that needs to be addressed is the claim made by Baise, following the lead of E.T. Jaynes, that men "ought" to assign the same probability to an event if they have the same "prior information." It is through this claim that Baise hopes to be able to bring objectivity in through the epistemological back door after having rejected it at the metaphysical front door. There are several grave flaws in this argument, however.

In the first place, it should be obvious that Baise's claim that men "ought" to assign the same probability to events is question-begging in this

context. *He is assuming that certain probabilities are objectively true in the course of trying to prove that they are objectively true.* This conclusion is unavoidable, because it would be completely nonsensical to say that men "ought" to believe something, *unless it is assumed from the outset to be objectively true.* Since this is precisely what Baise is attempting to prove, we have a patently question-begging argument on our hands. This error alone would be enough to doom the argument to failure.

This leads us to another serious problem with the argument. The focus on "prior information," as Baise and E.T. Jaynes put it, is something of a red herring in the context of the debate over the definition of probability. After all, the dispute between men like Richard von Mises and me does not concern "prior information." Indeed, even if Richard von Mises and I had *exactly* the same "prior information" about some future event, we would nevertheless continue to disagree vehemently about what methods are appropriate for generating a probability for the event. Richard von Mises would still dismiss all non-frequentist "prior information," whereas I would not. We would also continue to disagree with one another about the definition of probability, which is the central issue at hand. So, to argue that people who vehemently disagree with one another either would or "should" suddenly come together to embrace a single probability if they have the same "prior information" is not only ridiculously outlandish, but misses the central issue at hand.

An example can perhaps best illustrate this point. Suppose we want to find the probability of throwing a 2 with a die. There are many different methods we could utilize to come up with such a probability. We could use the so-called "classical method," and assume that each number on the die has an equal likelihood of being thrown, which would give us an *a priori* probability of 1/6. Or, we could use Richard von Mises and John Venn's relative frequency method and actually toss the die over and over in order to calculate the past relative frequency that a 2 is thrown. This number may or may not equal 1/6. Or, we could use some other method utilizing indirect information to come up with a probability that may or may not equal 1/6. Even if you and I have *all* of this prior information in front of both of us, we will probably still disagree about which number is "best," but Baise and Jaynes would have us believe that "two persons with the same prior information must assign the same probabilities." (Jaynes, 2005, p. 373). Well, which number "must" we assign? Neither Baise nor Jaynes informs us. What happens if we nevertheless disagree? Neither Baise nor Jaynes informs us. Nor, more importantly, do they tell us what probability is in the first place, which would allow us to determine which number, *if any*, is "best."

It is important to note, moreover, that this massive problem exists even in cases where people are assumed to have the same "prior information." The problem is multiplied exponentially in the real world where people rarely, if ever, have the same "prior information."

The Similarity of Baise's and Richard von Mises's Arguments

At a more abstract level, Baise is falling victim to the very same error that Richard von Mises succumbed to. As I argued in M. Crovelli (2009a), Richard von Mises made the mistake of attempting to elevate a particular *method* for generating numerical probabilities into a *definition* for probability. Baise is attempting to do something very similar with Bayes' Theorem: He is attempting to elevate and transform Bayes' Theorem into a definition for probability.

The main problem with the type of argument that Richard von Mises and Baise are making is that they are assuming that their favorite methods for generating numerical probabilities are in fact valid methods for generating probabilities before they even establish what probability *is*. The argument is circular, in other words. *Whether or not the relative frequency method, Bayesian methods, or any other methods are legitimate for generating probabilities depends upon the definition that we adopt for probability*. So, to assume from the outset that these methods are capable of generating numerical probabilities is to assume the very thing one is attempting to prove.

This error is blatantly obvious in Baise's case, since he is simply assuming that Bayesian methods are capable of generating valid numerical probabilities, despite the fact that dogmatic frequentists like Richard von Mises, for example, would passionately disagree.[3] The error is slightly less obvious in Richard von Mises's case, since virtually everyone agrees that the relative frequency method can generate probabilities, but it is nevertheless question-begging for him to assume that this is true *before* having a definition for probability in his hands.

In short, you must be able to say what probability *is* before you can say whether any particular method is actually measuring it, but both Richard von Mises and Arnold Baise simply take their favorite methods for granted, and then build their definitions for probability on this shaky foundation.

[3] In fact, Baise's argument is all the more circular since he is using a *theorem* of probability in an attempt to derive a definition for probability. I am grateful to Robert Crovelli for this point.

Conclusion

As I have argued elsewhere, the definition of probability is inextricably linked to the nature of the world. In a world where events and phenomena occur randomly and without cause, we are forced to adopt a different definition for probability than in a world where every event is caused. I have also argued that every event and phenomenon in the world *does in fact* have a cause of some sort, and this fact forces us to adopt a subjective definition for probability. That is, we must define probability as a measure of *our* uncertainty about the causal factors at work in the world.

This does not imply, however, that probabilities are meaningless. As our understanding of the causal factors at work in the world improves, our uncertainty simultaneously decreases, and the probabilities that we generate as a measure of our uncertainty become ever more accurate predictors of future events. This is not because probabilities are "objective" in any sense, as Baise and Richard von Mises claim, but because human uncertainty about the way the world works can be reduced over time.

References

Baise, Arnold. (2011) "Objective Bayesian Probability," *Libertarian Papers* 3, 20.

Crovelli, Mark R. (2009a). "On the Possibility of Assigning Probabilities to Singular Cases, or: Probability is Subjective Too!", *Libertarian Papers* 1, 26.

———. (2009b). 'Has David Howden Vindicated Richard von Mises's Definition of Probability?', *Libertarian Papers* 1, 44.

———. (2010). "A Challenge to Ludwig von Mises's Theory of Probability,"*Libertarian Papers* 2, 23.

———. (2011a). "Why the Definition of Probability Matters," *Mises Daily* (Jan. 26), mises.org/daily/4979/Why-the-Definition-of-Probability-Matters

———. (2011b). "In Defense of the Quants," *Mises Brasil Daily* (July 13), www.mises.org.br/ArticlePrint.aspx?id=1044

Crovelli, Robert A. (2011). "An Analysis of the Basic Concepts in Applied Mathematics," *Mises Working Paper Series*, mises.org/journals/scholar/crovelli4.pdf

Jaynes, E.T. (2005). *Probability Theory: The Logic of Science*, Cambridge.